Determination and correction of quadrature fringe measurement errors in interferometers

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The precision and accuracy of interferometers using quadrature fringe detection are often limited not by the interferometer itself but by the detector system. There are three typical errors: unequal gain in the two channels; quadrature phase shift error; and zero offsets. This paper describes a simple method for determining the quadrature errors from experimental data obtained in the interferometer and correcting for them. A numerical example demonstrating the significant improvement in the precision of interferometer data is given.

I. Introduction

All interferometers with quadrature detector systems have a set of errors in common, which in many cases severely limit the attainable precision and accuracy. These errors are (a) lack of quadrature (phase shift between reference signals is not exactly $\lambda/4$ or 90°), (b) unequal gain in the detector channels, and (c) zero offset. This paper describes a simple method of assessing these errors from experimental data and correcting for them. An ultrasonic fringe counting interferometer[1] is used as an example. The treatment is, however, quite general and is applicable to any interferometer with sinusoidal quadrature outputs.

II. Ultrasonic Interferometer

To illustrate the operation of idealized quadrature detector systems assume a wave train of carrier frequency f traversing a path length L in a medium with the propagation velocity c. If $u = \cos\omega t$ is the transmitted signal, the received signal is $u_r = R\cos(\omega t - \beta L)$, where $\omega = 2\pi f$, $\beta = 2\pi/\lambda = \omega/c$ is the wave number, λ is the wavelength, t is elapsed time, and R is an amplitude. The received signal is demodulated in a quadrature detector with the reference signals

$$r_1 = 2\cos\omega t \qquad r_2 = 2\cos(\omega t + \pi/2). \tag{1}$$

The output signals from the detector are

$$\begin{aligned} u_1 &= R[\cos\beta L + \cos(2\omega t - \beta L)],\\ u_2 &= R[\sin\beta L + \sin(2\omega t - \beta L)]. \end{aligned} \tag{2}$$

The second terms in Eqs. (2) are high frequency signals which can be suppressed with low-pass filters. The first terms are sinusoidal functions of path length L:

$$u_1 = R\cos\beta L \qquad u_2 = R\sin\beta L. \tag{3}$$

(u_1,u_2) describes a vector from the origin which rotates clockwise or counter clockwise depending on the sense of change of L. The end point of this vector is always on a circle of radius R.

Conventional techniques can be used to count the number of quarter wavelengths, fringes, traversed as L changes by sensing zero crossings of u_1 and u_2. Measurements of the instantaneous values of u_1 and u_2 permit interpolation between the counts by determining fractional fringes F which can be obtained from $\arctan u_2/u_1$. Problems caused by the multivalued nature of the arctan function can be avoided by using the expression

$$F = \tfrac{1}{2}\left|\frac{u_1}{|u_1|} - \frac{u_2}{|u_2|}\right| + 2\frac{u_1}{|u_1|}\cdot\frac{\arcsin u_2}{\pi}. \tag{4}$$

The change in path length ΔL in the interferometer can then be calculated from

$$\Delta L = [(\eta + F)_1 - (\eta + F)_0]\,\frac{\lambda}{4}, \tag{5}$$

where η is the integral fringe count, and subscripts 0 and 1 indicate measurements made before and after L was changed. The precision in determination of ΔL depends directly on the precision of measurement of n and F. With an ideal quadrature detector system the precison is often limited only by system noise. With real quadrature detectors, however, the end point of the vector (u_1,u_2) is rarely on a circle but on a distorted ellipse, and substantial errors arise in the fractional fringe F.

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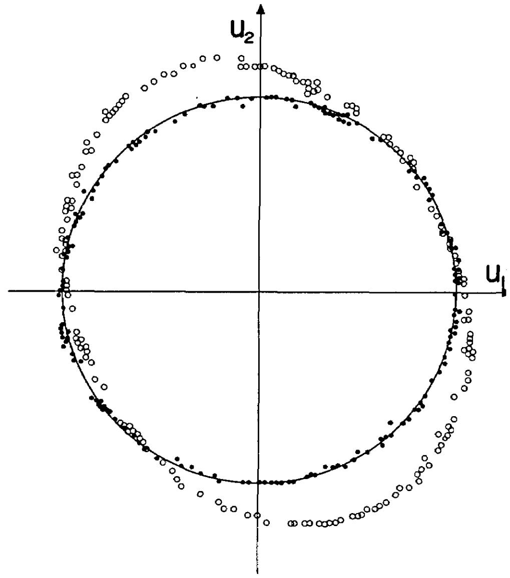

Fig. 1. Plots of uncorrected (○) and corrected (●) experimental data sets.

III. Quadrature Detector Errors

The distortion of the locus of the data points (u_1,u_2) from a circle into an ellipselike shape arises from three different effects which can be expressed as three coordinate transformations. The distorted ellipse $(u_1{}^d,u_2{}^d)$ is described by

$$u_1^d = u_1 + p,$$
$$u_2^d = \frac{1}{r}(u_2 \cos\alpha - u_1 \sin\alpha) + q, \tag{6}$$

where r is the channel gain ratio, p and q are the offset in the cosine and sine channels, and α is the reference signal quadrature error.

The original circle is now distorted into

$$(u_1 + p)^2 + \left(\frac{1}{r} u_2 \cos\alpha - \frac{1}{r} u_1 \sin\alpha + q\right)^2 = R^2, \tag{7}$$

where (u_1,u_2) are the signals that would be obtained from an ideal quadrature detector. Counts are no longer accumulated at $\sin\beta L = 0$ or at $\cos\beta L = 0$ but at

$$\cos\beta L + \frac{p}{R} = 0, \quad \beta L = \arccos\left(-\frac{p}{R}\right) + \left(m + \frac{1}{2}\right)\pi_1 \quad m = 0_1 1_1 \ldots, \tag{8}$$

$$\sin(\beta L + \alpha) + \frac{qr}{R} = 0, \quad \beta L = \arcsin\left(-\frac{qr}{R}\right) - \alpha + n\pi, \quad n = 0,1,\ldots. \tag{9}$$

The errors ϵ_i in the determination of L at the quadrant boundaries, where counts are accumulated are

$$\epsilon_1 = \frac{1}{\beta}\left[\arccos\left(-\frac{p}{R}\right) - \frac{\pi}{2}\right], \tag{10}$$

$$\epsilon_2 = \frac{1}{\beta}\left[\arcsin\left(-\frac{qr}{R}\right) - \alpha\right]. \tag{11}$$

To illustrate the magnitude of these errors, we calculated the errors ϵ_i relative to the wavelength λ for the following set of typical quadrature detector errors: $\alpha = -5°$; $p = q = 0.05$; $r = 1.1$; $R = 1$, and obtained

$$\frac{\epsilon_i}{\lambda} = \begin{cases} 0.004 \text{ near } \beta L = \pi/2, \\ 0.036 \text{ near } \beta L = 0. \end{cases}$$

Compared with the precision of good interferometers these are large errors. Similar errors are caused in the fractional fringes.

IV. Experimental Determination of Errors

If one assumes that p, q, r, and α are the only significant demodulation errors, a set of $(u_1{}^d,u_2{}^d)$ data taken over a sufficiently wide range of βL contains all the information needed to determine the four error terms. This can be achieved by fitting, in the least squares sense, Eq. (12), the equation of the distorted ellipse:

$$(u_1^d - p)^2 + \left[\frac{(u_2^d - q)r + (u_1^d - p)\sin\alpha}{\cos\alpha}\right]^2 = R^2 \tag{12}$$

in the following form:

$$Au_1^{d2} + Bu_2^{d2} + Cu_1^d u_2^d + Du_1^d + Eu_2^d = 1, \tag{13}$$

with

$$A = (R^2 \cos^2\alpha - p^2 - r^2q^2 - 2rpq \sin\alpha)^{-1},$$
$$B = Ar^2,$$
$$C = 2Ar \sin\alpha,$$
$$D = -2A(p + rq \sin\alpha),$$
$$E = -2Ar(rq + p \sin\alpha),$$

to the experimental data. Coefficients A through E and their standard deviations are obtained from this fitting operation. From the results, the quadrature errors and their random uncertainties σ_i are calculated:

$$\alpha = \arcsin C(4AB)^{-1/2} \tag{14}$$

and, as an example,

Table I. Quadrature Detector Correction Terms Obtained from Data Plotted in Fig. 1

A	0.932	δ_A	0.002	[Volt^{-2}]
B	0.686	δ_B	0.002	[Volt^{-2}]
C	0.311	δ_C	0.004	[Volt^{-2}]
D	−0.070	δ_D	0.002	[Volt^{-1}]
E	0.001	δ_E	0.002	[Volt^{-1}]
α	9.48	$d\alpha$	0.11	[degrees]
r	0.830	dr	0.002	
p	0.037	dp	0.001	[Volt]
q	0.01	dq	0.01	[Volt]

$$\frac{\delta\alpha}{\alpha} \approx \left[\left(\frac{1}{2}\frac{\delta_A}{A}\right)^2 + \left(\frac{1}{2}\frac{\delta_B}{B}\right)^2 + \left(\frac{1}{2}\frac{\delta_C}{C}\right)^2\right]^{1/2},$$

$$r = \left(\frac{B}{A}\right)^{1/2}, \tag{15}$$

$$p = \frac{2BD - EC}{C^2 - 4AB}, \tag{16}$$

$$q = \frac{2AE - DC}{C^2 - 4AB}. \tag{17}$$

V. Correction of Data

Knowing the quadrature errors experimental data $(u_1{}^d, u_2{}^d)$ taken with the interferometer can now be corrected using the inversion of Eq. (9):

$$u_1 = u_1^d - p;$$

$$u_2 = \frac{1}{\cos\alpha}\left[(u_1^d - p)\sin\alpha + r(u_2^d - q)\right]. \tag{18}$$

Figure 1 shows plots of experimental data from an ultrasonic interferometer before and after correction. The correction terms and their random uncertainties are given in Table I. After appropriate corrections are applied the typical noise-limited resolution of such interferometers is reduced to 10^{-8} m compared with 2×10^{-6} m for the uncorrected data for a carrier frequency of 10 MHz and a wavelength of 1.5×10^{-4} m.

Instead of varying the length of the path L to generate a data set for the extraction of the correction terms, either the carrier frequency f or the propagation velocity c could be varied. Care must be taken that new errors are not introduced, for example, due to limited receiver bandwidth or dispersion in the medium.

VI. Conclusions

Phase shift errors, zero offset, and gain differences may be found in most quadrature detectors. They may seriously affect the performance of the interferometers with which they are associated by increasing demodulation errors. The quadrature detector errors can easily be determined from experimental data, and suitable corrections can be made. This improves the resolution of many interferometers substantially.

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